

High-Frequency Reciprocity-Based Circuit Model for the Incidence of Electromagnetic Waves on General Waveguide Structures

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Abstract—In the present contribution we construct a high-frequency circuit model for the excitation of eigenmodes in general waveguides due to externally impinging electromagnetic waves. The circuit model, consisting of distributed sources in a transmission line model, is based on Lorentz's reciprocity theorem. The classical quasi-TEM solution of this problem is found as a special case from the full-wave model. The theory is illustrated with numerical examples of electric dipoles radiating above thick coupled lossy microstrip lines.

I. INTRODUCTION

AN IMPORTANT EMC-problem is the excitation of eigenmodes in waveguide structures due to externally impinging electromagnetic waves. This problem has been studied extensively in the past for thin wires above conducting grounds, i.e., for power distribution lines. As an entrance to literature we refer to [1]. In modern printed circuit boards and MMIC's the integration density and frequency or bitrate increase continuously. One of the resulting effects is the impingement of disturbing electromagnetic waves, due to nearby components, on interconnection lines. These interconnections, which are open waveguide structures, mostly take the form of multiconductor transmission lines (MTL's), however, the analysis presented below also includes dielectric waveguide structures.

In a previous publication [2] we put forward a general, new, and consistent circuit model for general waveguide structures and in particular MTL's valid in the full-wave regime. The purpose of the present paper is to show how an incident field impinging on these structures can be translated into source terms for this circuit model. In general, these sources will be expressed as per unit length distributed sources. If the incident field is spatially restricted to a particular longitudinal section of the structure (or to be more precise, if the incident field becomes negligibly small outside the considered section), we will introduce a lumped source typically located in the middle of the considered section.

The transmission line model developed in [2] describes the propagation of the discrete modes in the waveguide. Hence, the circuit model with sources will also only incorporate the effects of the excitation of the discrete modes. This means that surface waves or substrate waves and space waves which can

be excited by the impinging waves are not incorporated in the model [1], [3]. In other words, we assume that the excitation of these surface and space waves is local and that these waves, the amplitude of which decreases with the distance from the source, are negligible at the generator and load of the waveguide. Taking into account these surface waves and space waves requires a study of the detailed interaction of these waves with the load and generator.

In the past transmission line circuit models with sources have been constructed in the quasi-TEM limit [4]–[6]. In [7] the Lorentz reciprocity theorem is used to derive a circuit model for the incidence of a plane wave on a two-wire finite length transmission line in the quasi-TEM limit. The wires are placed in free space and the thin wire assumption is used. It is important to remark that [7] includes the effect of the terminations. Our approach is a full-wave one and comprises the well-documented quasi-TEM or low frequency approach as a special case. Our model incorporates general excitations and general geometries and is not restricted to plane wave excitations nor thin wires or perfect conductors as is often the case in literature.

The new model in [2] was based on the orthogonality of the eigenmodes and not on power assumptions. This makes the new model consistent even for lossy waveguides. Power related circuit models contain inconsistencies for lossy structures as has been shown in [2] and [8]. In the present paper we exploit the Lorentz reciprocity theorem to construct expressions for the sources in the circuit model. These sources are expressed as line integrals of the modal field distributions and the impinging fields. We remark that the determination of the sources does not require the full electromagnetic solution of the scattering of the impinging waves on the waveguide structure. For a MTL structure for example, first the eigenmodes of the MTL have to be determined followed by the scattering of the incident wave on the layered structure without the conductors.

We start with an introduction of the notations and a description of the geometry. Then we give a very short overview of the circuit model, developed in [2], needed for the further development of the theory. In the main body of the paper the integral expressions for the distributed sources are constructed. The final part of the theory consist of the derivation of the quasi-TEM limit from our full-wave model. This will yield the restrictions under which the quasi-TEM model is valid. The theory is illustrated with a distributed circuit model for

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electrical dipoles radiating above two coupled lossy microstrip lines.

II. GEOMETRY

We focus our attention to general MTL's with constant cross section consisting of an arbitrary number of N conductors embedded in a planar stratified medium. However, the developed theory is valid for general open waveguides such as dielectric waveguides and is not restricted to planar stratified media (see also [2]). A typical cross section is shown in Fig. 1 together with the coordinate system. The stratified medium rests on top of a perfectly conducting ground plane or terminates in a half-space. Each layer i ($i = 1, 2, \dots, L$) is homogeneous and isotropic with parameters ϵ_i and μ_i which can be complex to include losses or even gain. Each of the N conductors is either perfectly conducting, exhibits a finite conductivity or could even be a pure dielectric waveguide. In this last case the notion "conductor" must be understood in a generalized sense. We restrict our analysis to the frequency domain. The common time dependence $e^{j\omega t}$ is suppressed throughout the paper. An arbitrary incident wave [$\mathbf{e}^{inc}(x, y, z)$, $\mathbf{h}^{inc}(x, y, z)$] impinges on the MTL from the upper half-space (typically free space).

III. TRANSMISSION LINE MODEL FOR A GENERAL WAVEGUIDE

We now focus our attention to a longitudinal section of the MTL located between the planes $z = z_1$ and $z = z_2$ (see Fig. 1). As a starting point of our calculations we represent the fields along that section in terms of a finite number C of discrete eigenmodes. We assume that higher order modes are less important because they are strongly damped due to losses or because they are in cut-off. The number C depends on the frequency but for a typical MTL application the fundamental eigenmodes will often suffice, although our theory is valid for the other modes too (see also [2]). There are N fundamental modes in the presence of a ground plane ($C = N$) or $N-1$ if no ground plane is present ($C = N-1$). Hence an arbitrary (e, h) field propagating along the MTL can be represented by

$$\begin{aligned} \mathbf{e}(x, y, z) &= \sum_{f=1}^C K_f^+ e^{-j\beta_f z} (\mathbf{E}_{tr, f} + E_{z, f} \mathbf{u}_z) \\ &\quad + \sum_{f=1}^C K_f^- e^{+j\beta_f z} (\mathbf{E}_{tr, f} - E_{z, f} \mathbf{u}_z) \\ \mathbf{h}(x, y, z) &= \sum_{f=1}^C K_f^+ e^{-j\beta_f z} (\mathbf{H}_{tr, f} + H_{z, f} \mathbf{u}_z) \\ &\quad - \sum_{f=1}^C K_f^- e^{+j\beta_f z} (\mathbf{H}_{tr, f} - H_{z, f} \mathbf{u}_z). \end{aligned} \quad (1)$$

$\mathbf{E}_{tr, f}(x, y)$, $\mathbf{H}_{tr, f}(x, y)$, $E_{z, f}(x, y)$, and $H_{z, f}(x, y)$ are the transversal and longitudinal eigenmode patterns obtained from an eigenmode analysis (see e.g., [9], [10]). They only depend upon the transversal coordinates x and y . K_f^+ and K_f^- are the excitation coefficients of waves travelling in the positive and negative z -direction respectively, β_f is the eigenvalue or

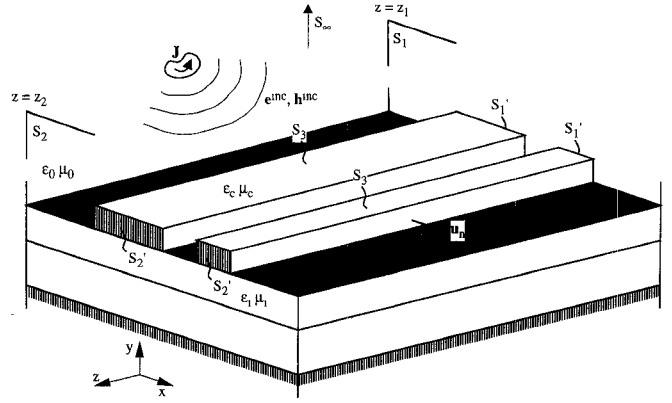


Fig. 1. A typical multiconductor transmission line between the planes $z = z_1$ and $z = z_2$.

propagation factor of mode f . One can prove the eigenmodes to be orthogonal [2]. We select them to be orthonormal such that

$$\frac{1}{2} \iint_{S_c} (\mathbf{E}_{tr, f} \times \mathbf{H}_{tr, g}) \cdot \mathbf{u}_z dS = \delta_{fg} \quad f, g = 1, 2, \dots, C \quad (2)$$

where $\delta_{fg} = 1$ for $f = g$, where $\delta_{fg} = 0$ for $f \neq g$ and where S_c is the total cross section of the MTL. The values of the coefficients K_f^+ and K_f^- are determined by the sources and loads at the beginning and the end of the MTL. In [2] it is shown that the following circuit model is a suitable representation of the wave phenomena (1)

$$\begin{aligned} \underline{V}(z) &= 2(\underline{I}_m^T)^{-1} e^{-j\beta z} \underline{K}^+ + 2(\underline{I}_m^T)^{-1} e^{j\beta z} \underline{K}^- \\ \underline{I}(z) &= \underline{I}_m e^{-j\beta z} \underline{K}^+ - \underline{I}_m e^{j\beta z} \underline{K}^-. \end{aligned} \quad (3)$$

\underline{V} and \underline{I} are column vectors the C elements of which are the voltages and currents of the circuit model. \underline{K}^+ and \underline{K}^- are column vectors with elements K_f^+ and K_f^- , respectively and $\underline{\beta}$ is a diagonal matrix with the β_f -values on its diagonal ($f = 1, 2, \dots, C$). \underline{I}_m is a $C \times C$ matrix the columns of which are the current eigenvectors of the circuit model. The transmission line model (3) can also be described by the following set of Telegrapher's equations

$$\begin{aligned} \frac{d\underline{V}}{dz} + \underline{Z}\underline{I} &= 0 \\ \frac{d\underline{I}}{dz} + \underline{Y}\underline{V} &= 0 \end{aligned} \quad (4)$$

with

$$\begin{aligned} \underline{Z} &= j\omega \underline{L} + \underline{R} = 2j(\underline{I}_m^T)^{-1} \underline{\beta} (\underline{I}_m)^{-1} \\ \underline{Y} &= j\omega \underline{C} + \underline{G} = \frac{j}{2} \underline{I}_m \underline{\beta} \underline{I}_m^T. \end{aligned} \quad (5)$$

\underline{L} , \underline{R} , \underline{C} , and \underline{G} are the (frequency dependent) inductance, resistance, capacitance and conductance $C \times C$ matrices. The $C \times C$ characteristic impedance matrix of the transmission line model is given by $\underline{Z}_{char} = 2(\underline{I}_m^T)^{-1} (\underline{I}_m)^{-1}$. The mapping of the wave phenomena (1) onto the circuit model (3)-(5) depends on the choice of \underline{I}_m . We refer the reader to the

detailed discussions in [2] and [4]. For MTL's the most adopted definition for the elements of \underline{L}_m is [11]

$$I_{m,jf} = \oint_{c_j} \mathbf{H}_{tr,f} \cdot d\mathbf{l} \quad j, f = 1, 2, \dots, C \quad (6)$$

where c_j is the circumference of conductor j . This means that $I_{m,jf}$ is the total current through conductor j due to eigenmode f . This definition is used in the power-current impedance definition for microstrip and stripline problems [11]. At low frequencies, i.e., in the quasi-TEM limit and in the absence of losses, \underline{R} and \underline{G} are zero and \underline{L} and \underline{C} become frequency independent and take their classical meaning ([12], [13]).

IV. INTRODUCTION OF SOURCE TERMS STARTING FROM A RECIPROCIITY THEOREM

We now want to introduce the effect of the incident wave ($\mathbf{e}^{inc}, \mathbf{h}^{inc}$) into the circuit model. This will amount to determining the values of the voltage and current sources that must be introduced at the right-hand side of (4) in order to account for the influence of the incident wave.

As a first step, we remove the conductors from the problem. We then calculate the diffracted field ($\mathbf{e}^d, \mathbf{h}^d$) due to the presence of the planar stratified medium. In the sequel only the ($\mathbf{e}^t = \mathbf{e}^{inc} + \mathbf{e}^d, \mathbf{h}^t = \mathbf{h}^{inc} + \mathbf{h}^d$) fields will be of importance. If we now again introduce the conductors a scattered field ($\mathbf{e}^{sc}, \mathbf{h}^{sc}$) is generated outside the conductors. Hence, the total field outside the conductors is given by ($\mathbf{e}^{inc} + \mathbf{e}^d + \mathbf{e}^{sc}, \mathbf{h}^{inc} + \mathbf{h}^d + \mathbf{h}^{sc}$). The total field inside the conductors is denoted by ($\mathbf{e}^{cond}, \mathbf{h}^{cond}$).

To explain the approach that will be adopted, consider the example of two coupled microstrip lines with rectangular cross section located between $z = z_1$ and $z = z_2$ as shown on Fig. 1. Note that we explicitly consider penetrable conductors with arbitrary complex material parameters ε_c and μ_c , which can also be different for each conductor. The conductivity of the conductors or the imaginary part of ε_c can vary from zero (dielectric waveguide) to infinity (perfect conductor). In the sequel we will apply the following reciprocity theorem [14]. If two sets of fields ($\mathbf{e}_a, \mathbf{h}_a$), and ($\mathbf{e}_b, \mathbf{h}_b$) satisfy Maxwell's equations in the same sourceless volume, the following relationship holds on the boundary surface S of that volume

$$\iint_S (\mathbf{e}_a \times \mathbf{h}_b - \mathbf{e}_b \times \mathbf{h}_a) \cdot \mathbf{u}_n dS = 0. \quad (7)$$

In (7) \mathbf{u}_n is the unit normal vector to S . We consider two volumes (see Fig. 1). The "outside" volume R_{out} is the volume bounded by the surface S_1 which corresponds to the plane $z = z_1$ except for the S_1' surface of the microstrip cross section, the surface S_2 (the $z = z_2$ plane minus S_2'), the ground plane, a surface part located at infinity S_∞ and the surface S_3 , consisting of four parts on each microstrip (the bottom, top, and side surfaces of the strips). The "inside" volume R_{in} corresponds to the inside of the microstrips with surfaces $S_3, S_1',$ and S_2' . We now apply (7) to R_{out} where the a -field is the scattered field ($\mathbf{e}^{sc}, \mathbf{h}^{sc}$) and the b -field is the

field of one particular mode [normalized with (2)] propagating in the positive z -direction. Note that the subscript f in (1) can take the values 1 and 2 in the considered two strip example. From (1) and (2) such a modal field is seen to be

$$\begin{aligned} \mathbf{e}_k &= e^{-j\beta_k z} (\mathbf{E}_{tr,k} + E_{z,k} \mathbf{u}_z) \\ \mathbf{h}_k &= e^{-j\beta_k z} (\mathbf{H}_{tr,k} + H_{z,k} \mathbf{u}_z) \end{aligned} \quad (8)$$

where k can be either 1 or 2 in our example. We also apply (7) to R_{in} . The b -field remains the same as for R_{out} , but the a -field is now replaced by ($\mathbf{e}^{cond}, \mathbf{h}^{cond}$). (7) then leads to the following results for R_{out} and R_{in} , respectively

$$\begin{aligned} \iint_{S_1+S_2+S_3} (\mathbf{e}^{sc} \times \mathbf{h}_k - \mathbf{e}_k \times \mathbf{h}^{sc}) \cdot \mathbf{u}_n dS &= 0 \\ \iint_{S_1'+S_2'+S_3} (\mathbf{e}^{cond} \times \mathbf{h}_k - \mathbf{e}_k \times \mathbf{h}^{cond}) \cdot \mathbf{u}_n dS &= 0. \end{aligned} \quad (9)$$

Note that in (9) we already left out the contributions coming from the ground plane and the contributions coming from S_∞ . The former ones are zero because ($\mathbf{u}_n \times \mathbf{e}^{sc}$) and ($\mathbf{u}_n \times \mathbf{e}_k$) are zero on a perfect conductor. The latter ones drop out as a consequence of the radiation conditions satisfied by the propagating mode field and by the scattered field. We can think of both the scattered field and the modal field as being generated by equivalent electric and magnetic sources on the conductor surfaces. As these sources have a limited extent, the scattered field will automatically satisfy the radiation conditions. We now invoke the following boundary conditions valid on S_3

$$\begin{aligned} \mathbf{u}_n \times \mathbf{e}^{cond} &= \mathbf{u}_n \times \mathbf{e}^{sc} + \mathbf{u}_n \times \mathbf{e}^t \\ &= \mathbf{u}_n \times \mathbf{e}^{sc} + \mathbf{u}_n \times \mathbf{e}^{inc} + \mathbf{u}_n \times \mathbf{e}^d \\ \mathbf{u}_n \times \mathbf{h}^{cond} &= \mathbf{u}_n \times \mathbf{h}^{sc} + \mathbf{u}_n \times \mathbf{h}^t \\ &= \mathbf{u}_n \times \mathbf{h}^{sc} + \mathbf{u}_n \times \mathbf{h}^{inc} + \mathbf{u}_n \times \mathbf{h}^d. \end{aligned} \quad (10)$$

Subtraction of the two equations in (9), combined with (10) finally leads to

$$\begin{aligned} - \iint_{z=z_1} (\mathbf{e}_a \times \mathbf{h}_k - \mathbf{e}_k \times \mathbf{h}_a) \cdot \mathbf{u}_z dS \\ + \iint_{z=z_2} (\mathbf{e}_a \times \mathbf{h}_k - \mathbf{e}_k \times \mathbf{h}_a) \cdot \mathbf{u}_z dS \\ = - \iint_{S_3} (\mathbf{e}^t \times \mathbf{h}_k - \mathbf{e}_k \times \mathbf{h}^t) \cdot \mathbf{u}_n dS. \end{aligned} \quad (11)$$

In (11), the integrals on the left hand side extend over the total cross section of the structure at $z = z_1$ and $z = z_2$, respectively, while the integral in the right-hand side extends over the outer surface of the microstrips for $z_1 < z < z_2$. \mathbf{u}_n is pointing outward the conductors (see Fig. 1). The field ($\mathbf{e}_a, \mathbf{h}_a$) is ($\mathbf{e}^{sc}, \mathbf{h}^{sc}$) on S_1 and S_2 , i.e., outside the conductors, and ($\mathbf{e}^{cond}, \mathbf{h}^{cond}$) on S_1' and S_2' , i.e., inside the conductors.

The $(\mathbf{e}_a, \mathbf{h}_a)$ field is entirely due to the presence of the conductors. This field will excite modal fields on the conductor, propagating modes as well as nonpropagating ones.

Until now no assumptions were made to obtain (11). If we now suppose that the $(\mathbf{e}^i, \mathbf{h}^i)$ field is negligible for $z > z_2$ and for $z < z_1$ and that at $z = z_1$ and $z = z_2$ the effect of the nonpropagating modes has already died out, the transversal part of the $(\mathbf{e}_a, \mathbf{h}_a)$ field can be written as [see (1)]

$$\begin{aligned} \mathbf{e}_{a, tr} &= \sum_{f=1}^C P_f e^{-j\beta_f z} \mathbf{E}_{tr, f} \\ \mathbf{h}_{a, tr} &= \sum_{f=1}^C P_f e^{-j\beta_f z} \mathbf{H}_{tr, f} \end{aligned} \quad (12)$$

for $z \geq z_2$ and

$$\begin{aligned} \mathbf{e}_{a, tr} &= \sum_{f=1}^C Q_f e^{+j\beta_f z} \mathbf{E}_{tr, f} \\ \mathbf{h}_{a, tr} &= - \sum_{f=1}^C Q_f e^{+j\beta_f z} \mathbf{H}_{tr, f} \end{aligned} \quad (13)$$

for $z \leq z_1$ and with $f = 1, 2$ (i.e., $C = 2$) in our particular example. As a final step we insert (12) and (13) into (11) and using the orthogonality relationships (2) we arrive at

$$\begin{aligned} Q_k &= \frac{1}{4} \iint_{S_3} [(\mathbf{u}_n \times \mathbf{e}^i) \cdot (\mathbf{H}_{tr, k} + H_{z, k} \mathbf{u}_z) \\ &\quad + (\mathbf{u}_n \times \mathbf{h}^i) \cdot (\mathbf{E}_{tr, k} + E_{z, k} \mathbf{u}_z)] e^{-j\beta_k z} dS \end{aligned} \quad (14)$$

with $k = 1, 2, \dots, C$. If we now replace the $(\mathbf{e}_b, \mathbf{h}_b)$ field used in the above reasoning [i.e., the modal field (8)] by the normalized modal field propagating in the negative z -direction [obtained by replacing β_k by $-\beta_k$, $\mathbf{H}_{tr, k}$ by $-\mathbf{H}_{tr, k}$, and $E_{z, k}$ by $-E_{z, k}$ in (8)] we can conclude that

$$\begin{aligned} P_k &= \frac{1}{4} \iint_{S_3} [(\mathbf{u}_n \times \mathbf{e}^i) \cdot (-\mathbf{H}_{tr, k} + H_{z, k} \mathbf{u}_z) \\ &\quad + (\mathbf{u}_n \times \mathbf{h}^i) \cdot (\mathbf{E}_{tr, k} - E_{z, k} \mathbf{u}_z)] e^{+j\beta_k z} dS. \end{aligned} \quad (15)$$

Equations (12)–(15) show how the field $(\mathbf{e}^i, \mathbf{h}^i)$, which is itself the combined effect of the incident field and the diffraction by the planar stratified medium, excites propagating modes to the right and to the left of this part of the MTL along which the $(\mathbf{e}^i, \mathbf{h}^i)$ field exists. Only those field components of $(\mathbf{e}^i, \mathbf{h}^i)$ tangential to the conductor surfaces come into play, projected on particular tangential components of the modal patterns. The integration in (14) and (15) falls into two parts: one integration is a simple integration over the circumferences of the cross sections of each conductor, the other integration amounts to a spatial Fourier transformation along the z -axis for the Fourier variables $-\beta_k$ and β_k , respectively. As the modal patterns are z -independent, only the $(\mathbf{e}^i, \mathbf{h}^i)$ field is Fourier transformed.

Before we turn to the circuit model, we look at the important special case of perfect conductors ($\sigma_c = \infty$ or $\varepsilon_c = -j\infty$). In

that case $\mathbf{u}_n \times (\mathbf{E}_{tr, k} - E_{z, k} \mathbf{u}_z)$ in (14) and (15) is zero. Hence

$$\begin{aligned} Q_k &= -\frac{1}{4} \iint_{S_3} \mathbf{e}^i \cdot \mathbf{J}_{s, k}^+ e^{-j\beta_k z} dS \\ P_k &= -\frac{1}{4} \iint_{S_3} \mathbf{e}^i \cdot \mathbf{J}_{s, k}^- e^{+j\beta_k z} dS. \end{aligned} \quad (16)$$

We have introduced the modal surface currents $\mathbf{J}_{s, k}^+$ and $\mathbf{J}_{s, k}^-$ of mode k for propagation in the positive (superscript +) respectively negative (superscript -) z -direction. Their z -components are identical, their transversal components differ in sign.

We draw the attention of the reader to the fact that we have explicitly assumed that the incident field is negligible outside $z_1 < z < z_2$ and that if this is not the case, care should be taken to include the interaction with terminations outside $z_1 < z < z_2$ (i.e., the load and generator). We refer to [7] for a way in which this can be done based on Lorentz reciprocity for the situation of a rectangular thin wire loop in free space.

V. CIRCUIT MODEL FOR THE MULTICONDUCTOR TRANSMISSION LINE IN AN INCIDENT FIELD

In this section we will show that the field calculation results (14) and (15) can be expressed in circuit terms by introducing source terms at the right-hand side of (4). The simplest way to prove this is to first introduce source terms $\underline{v}_g \delta(z - z')$ and $\underline{i}_g \delta(z - z')$ at the right-hand side of the first and second equation in (4) respectively, where \underline{v}_g and \underline{i}_g are $C \times 1$ column vectors and to calculate the corresponding solution of the resulting differential equations. This solution turns out to be

$$\begin{aligned} \underline{V}(z) &= -\frac{1}{2} (\underline{I}_m^T)^{-1} e^{+j\beta(z-z')} (\underline{I}_m^T \underline{v}_g - 2\underline{I}_m^{-1} \underline{i}_g) \\ \underline{I}(z) &= \frac{1}{4} (\underline{I}_m) e^{+j\beta(z-z')} (\underline{I}_m^T \underline{v}_g - 2\underline{I}_m^{-1} \underline{i}_g) \end{aligned} \quad (17)$$

for $z < z'$ and

$$\begin{aligned} \underline{V}(z) &= \frac{1}{2} (\underline{I}_m^T)^{-1} e^{-j\beta(z-z')} (\underline{I}_m^T \underline{v}_g + 2\underline{I}_m^{-1} \underline{i}_g) \\ \underline{I}(z) &= \frac{1}{4} (\underline{I}_m) e^{-j\beta(z-z')} (\underline{I}_m^T \underline{v}_g + 2\underline{I}_m^{-1} \underline{i}_g) \end{aligned} \quad (18)$$

for $z > z'$, as can be readily checked. Comparing (17) and (18) to (3), we see that (17) and (18) are of the form (3) with

$$\underline{K}^+ = \frac{1}{4} e^{+j\beta z'} (\underline{I}_m^T \underline{v}_g + 2\underline{I}_m^{-1} \underline{i}_g)$$

and

$$\underline{K}^- = 0 \quad (19)$$

for $z > z'$ and with

$$\underline{K}^- = -\frac{1}{4} e^{-j\beta z'} (\underline{I}_m^T \underline{v}_g - 2\underline{I}_m^{-1} \underline{i}_g)$$

and

$$\underline{K}^+ = 0 \quad (20)$$

for $z < z'$. In Section III we explained how the field representations (1) are translated into the circuit representation (3), where the coefficients \underline{K}^+ and \underline{K}^- are identical in both (1) and (3). Starting from this observation and from (19) and (20), we can conclude that the sources $\underline{v}_g \delta(z - z')$ and $\underline{i}_g \delta(z - z')$

will be the circuit representation of field sources giving rise to the following transversal electric fields

$$\begin{aligned} \mathbf{e}_{tr} &= \sum_{f=1}^C K_f^+ e^{-j\beta_f z} \mathbf{E}_{tr,f} \quad \text{for } z > z' \\ \mathbf{e}_{tr} &= \sum_{f=1}^C K_f^- e^{+j\beta_f z} \mathbf{E}_{tr,f} \quad \text{for } z < z' \end{aligned} \quad (21)$$

with K_f^+ the elements of the column vector \underline{K}^+ in the first equation of (19) and with K_f^- the elements of the column vector \underline{K}^- in the second equation of (19). Similar equations can of course be written down for the magnetic field.

If we now consider the following circuit equations:

$$\begin{aligned} \frac{dV}{dz} + \underline{Z}I &= \underline{V}_g(z) \\ \frac{dI}{dz} + \underline{Y}V &= \underline{I}_g(z) \end{aligned} \quad (22)$$

where $\underline{V}_g(z)$ and $\underline{I}_g(z)$ only differ from zero between z_1 and z_2 , it follows from (20) that $\underline{V}_g(z)$ and $\underline{I}_g(z)$ will be the circuit representation of field sources giving rise to the following transversal electric field:

$$\begin{aligned} \mathbf{e}_{tr} &= \sum_{f=1}^C A_f e^{-j\beta_f z} \mathbf{E}_{tr,f} \quad \text{for } z > z_2 \\ \mathbf{e}_{tr} &= \sum_{f=1}^C B_f e^{+j\beta_f z} \mathbf{E}_{tr,f} \quad \text{for } z < z_1 \end{aligned} \quad (23)$$

with

$$\begin{aligned} \underline{A} &= \int_{z_1}^{z_2} \frac{1}{4} e^{+j\beta z'} [\underline{I}_m^T \underline{V}_g(z') + 2(\underline{I}_m)^{-1} \underline{I}_g(z')] dz' \\ \underline{B} &= - \int_{z_1}^{z_2} \frac{1}{4} e^{-j\beta z'} [\underline{I}_m^T \underline{V}_g(z') - 2(\underline{I}_m)^{-1} \underline{I}_g(z')] dz'. \end{aligned} \quad (24)$$

\underline{A} and \underline{B} are again column vectors with A_f and B_f as elements. It now suffices to compare (23) to (12) and (13), to see that to account for the incident fields studied in the previous section, A_f must be identified with P_f and B_f with Q_f . With (14) and (15) this leads to the final values of $\underline{V}_g(z)$ and $\underline{I}_g(z)$

$$\begin{aligned} \underline{V}_g(z) &= -(\underline{I}_m^T)^{-1} \underline{p}(z) \\ \underline{p}(z) &= \oint_{c_3} [(\mathbf{u}_n \times \mathbf{e}^i) \cdot \underline{\mathbf{H}}_{tr} + (\mathbf{u}_n \times \mathbf{h}^i) \cdot \underline{E}_z \mathbf{u}_z] dc \\ \underline{I}_g(z) &= \frac{1}{2} \underline{I}_m \underline{g}(z) \\ \underline{g}(z) &= \oint_{c_3} [(\mathbf{u}_n \times \mathbf{e}^i) \cdot \underline{H}_z \mathbf{u}_z + (\mathbf{u}_n \times \mathbf{h}^i) \cdot \underline{\mathbf{E}}_{tr}] dc. \end{aligned} \quad (25)$$

Note that in (25) only \mathbf{e}^i and \mathbf{h}^i are z -dependent. We have grouped the transversal and longitudinal eigenmode field components into $C \times 1$ column vectors. $\underline{\mathbf{E}}_{tr}$, e.g., consists of elements $\mathbf{E}_{tr,f}(x, y)$ with $f = 1, 2, \dots, C$. The integration in (25) now only extends over the circumferences c_3 of the cross sections of each conductor. The Fourier transformation along the z -axis in (14) and (15) will now automatically emerge from the solution of (22).

From the reasoning put forward in the previous section and in particular because only propagating modes are taken into account, it must be clear to the reader that strictly speaking the particular solution to (22) only makes sense for $z \leq z_1$ and for $z \geq z_2$. To emphasize this fact one could also replace $\underline{V}_g(z)$ and $\underline{I}_g(z)$ by two lumped sources \underline{V}_{gl} and \underline{I}_{gl} , e.g., located at z_0 ($z_1 \leq z_0 \leq z_2$)

$$\begin{aligned} \underline{V}_{gl}(z) &= \delta(z - z_0) (\underline{I}_m^T)^{-1} \int_{z_1}^{z_2} \{ \cos[\underline{\beta}(z' - z_0)] \underline{p}(z') \\ &\quad + j \sin[\underline{\beta}(z' - z_0)] \underline{g}(z') \} dz' \\ \underline{I}_{gl}(z) &= \frac{1}{2} \delta(z - z_0) \underline{I}_m \int_{z_1}^{z_2} \{ \cos[\underline{\beta}(z' - z_0)] \underline{g}(z') \\ &\quad - j \sin[\underline{\beta}(z' - z_0)] \underline{p}(z') \} dz'. \end{aligned} \quad (26)$$

Both (25) and (26) show that, generally speaking, even for perfect conductors both a voltage and a current source have to be introduced.

A last point we want to deal with in this section is the special case of an incident plane wave. Typical calculations [4]–[6] start from the assumption that the plane wave is present from $z = -\infty$ to $z = +\infty$. From this a distributed source representation is derived and in the circuit calculations these distributed sources are then restricted to a finite length of coupled transmission lines. In our approach of the problem this amounts to inserting that plane wave into $\underline{p}(z)$ and $\underline{g}(z)$ and disregarding the requirement that $(\mathbf{e}^i, \mathbf{h}^i)$ should vanish outside the (z_1, z_2) interval and thus neglecting the end effects occurring in the neighborhood of $z = z_1$ and $z = z_2$ and neglecting the effect of that plane wave on the terminals. If $(\mathbf{e}^{inc}, \mathbf{h}^{inc})$ is a plane wave, that wave will have a z -dependence of the form $e^{jk_z z}$. This will automatically also be the case for $(\mathbf{e}^d, \mathbf{h}^d)$ and $(\mathbf{e}^i, \mathbf{h}^i)$. This in turn implies that $\underline{p}(z)$ and $\underline{g}(z)$ can be rewritten as $e^{jk_z z} \underline{A}$ and $e^{jk_z z} \underline{B}$ where \underline{A} and \underline{B} are now constant, i.e., z -independent vectors and that the $e^{jk_z z}$ -dependence also shows up in \underline{V}_g and \underline{I}_g . Looking at (26) it is clear that \underline{V}_{gl} and \underline{I}_{gl} can also be expressed in terms of \underline{A} and \underline{B} and that the integration with respect to z can be done analytically. We leave this to the reader.

VI. APPLICATION TO THE QUASI-TEM CASE

In this section, we will restrict ourselves to the case of perfect conductors, to lossless dielectrics and to the low-frequency region in which the fundamental modes used in (1) are quasi-TEM modes. It is our purpose to show that the general result (25) reduces to published results for the quasi-TEM limit.

In the quasi-TEM limit the longitudinal eigenmode fields $E_{z,f}(x, y)$ and $H_{z,f}(x, y)$ are negligible with respect to the transversal ones which can be derived from suitable scalar functions [13]

$$\begin{aligned} \mathbf{E}_{tr,f} &= -\nabla_{tr} \phi_f(x, y) \\ \mathbf{H}_{tr,f} &= -\frac{1}{\mu} \mathbf{u}_z \times \nabla_{tr} \psi_f(x, y). \end{aligned} \quad (27)$$

ϕ_f has the meaning of a potential, ψ_f on the other hand has the meaning of a magnetic flux [13]. They are both zero on the reference conductor and take a constant value

on the other conductors. The gradient operator ∇_{tr} only operates on the transversal coordinates x and y and μ takes a different value $\mu = \mu_i$ in each layer of the stratified medium. If we now consider two modes f and g with corresponding functions ϕ_f, ψ_f and ϕ_g, ψ_g we first impose the orthonormality condition (2)

$$\frac{1}{2} \iint_{S_c} \frac{1}{\mu} \nabla_{tr} \phi_f \cdot \nabla_{tr} \psi_g dS = \delta_{fg} \quad f, g = 1, 2, \dots, C. \quad (28)$$

Using a Green's theorem, (28) can be rewritten as

$$-\frac{1}{2} \int_c \frac{\phi_f}{\mu} \frac{\partial \psi_g}{\partial n} dc = \delta_{fg} \quad (29)$$

where we used the fact that ψ_g satisfies Laplace's equation. The integration extends over the circumferences of the cross sections of all conductors and $\partial/\partial n$ is taken with respect to the outward pointing normal \mathbf{u}_n . As ϕ_f and $(1/\mu)(\partial\psi_g/\partial n)$ are continuous throughout the cross-section (the first quantity being a potential, the second one a tangential magnetic field component), (29) does not contain contributions from layer interfaces. Replacing ϕ_f by its constant value V_{fi} on each conductor i ($i = 1, 2, \dots, C$) and noting that $(1/\mu)(\partial\psi_g/\partial n) = -H_c$, where H_c is the tangential component of the magnetic field at each conductor (with $\mathbf{u}_c = -\mathbf{u}_n \times \mathbf{u}_z$), (29) reduces to

$$\frac{1}{2} \sum_{i=1}^C V_{fi} I_{gi} = \frac{1}{2} \underline{V}_f^T \underline{I}_g = \delta_{fg}. \quad (30)$$

The summation extends over all conductors and I_{gi} is the total current flowing through conductor i for mode g . For further use below we have grouped voltages and currents into two $C \times 1$ column vectors. A simple choice to satisfy (30) would, e.g., be $V_{fi} = \delta_{fi}$ and $I_{gi} = 2\delta_{gi}$. This implies that mode f would correspond with zero potential on each conductor except for conductor f where the potential is 1 volt. Mode g then corresponds with zero current flowing through each conductor except for conductor g which supports a current of 2 ampere. This choice would be possible in the pure static case ($\omega = 0$) where the electric and magnetic field problem are completely decoupled. However, in the quasi-static case both problems are coupled and conductor voltages and currents are found to be related. In [13] it is shown that the eigenvoltages and eigencurrents of the quasi-TEM problem satisfy the following equations:

$$\begin{aligned} \beta \underline{v} &= \pm \omega \underline{L} \underline{i} \\ \beta \underline{i} &= \pm \omega \underline{C} \underline{v} \end{aligned} \quad (31)$$

with the plus (minus) sign for propagation in the positive (negative) z -direction and hence

$$\begin{aligned} \omega^2 (\underline{L} \underline{C}) \underline{v} &= \beta^2 \underline{v} \\ \omega^2 (\underline{C} \underline{L}) \underline{i} &= \beta^2 \underline{i} \end{aligned} \quad (32)$$

where \underline{v} and \underline{i} are a set of corresponding eigenvoltages and eigencurrents, and where β is the corresponding eigenvalue. If we now choose \underline{V}_f to be equal to one of the C solutions of the first equation in (32) and \underline{I}_g to be equal to one of the

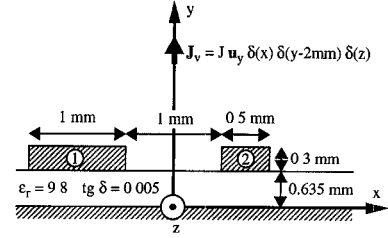


Fig. 2. Geometry of two coupled asymmetric thick microstrip lines on a lossy substrate excited by a vertical dipole $\mathbf{J}_v = J \mathbf{u}_y \delta(x) \delta(y-2\text{mm}) \delta(z)$.

C solutions of the second equation in (32), one can easily prove that $\underline{V}_f^T \underline{I}_g = 0$ for $f \neq g$ and by a suitable selection of the amplitudes of the eigenvectors it is possible to arrive at $\underline{V}_f^T \underline{I}_f = 2$ for all f . The C eigenvalues β of (32), which are identical for both equations, of course correspond to the propagation factors β_f used in (1).

We now turn to (25). For perfect conductors the second term in both \underline{p} and \underline{g} is identically zero. The first term in \underline{g} can also be neglected as it depends upon a longitudinal eigenmode field. In the quasi-TEM limit, which is in fact a low-frequency approximation, \mathbf{e}^z can approximately be regarded to be constant over the cross section of each conductor. Taking this constant value outside the integration, it can easily be verified that the integration reduces to the integration of $-(\mathbf{H}_{tr} \cdot \mathbf{u}_c)$ over the circumference of each conductor. In conclusion, it is found that

$$\begin{aligned} p_f(z) &= - \sum_{i=1}^C e_{z,i}^z(z) I_{fi} \\ g_f(z) &= 0 \end{aligned} \quad (33)$$

where I_{fi} is the value of the eigencurrent of mode f on conductor i , as already defined above and where $e_{z,i}^z(z)$ is the z -component of \mathbf{e}^z on conductor i . To finally determine \underline{V}_g we first turn back to (6). The column index f in $I_{m,jf}$ refers to mode f while the row index j refers to the conductor. This implies that $I_{m,jf} = I_{fj}$ and that the rows of the $C \times C$ matrix \underline{I}_m are given by $(\underline{I}_f)^T$. From (25) we find that $\underline{p}(z) = -(\underline{I}_m)^T \underline{V}_g$ or in component form and taking into account (33)

$$\begin{aligned} p_f(z) &= - \sum_{i=1}^C e_{z,i}^z(z) I_{fi} \\ &= - \sum_{i=1}^C V_{g,i}(z) I_{fi} \end{aligned} \quad (34)$$

leading to the conclusion that element f of the column vector \underline{V}_g is given by $e_{z,i}^z(z)$ and that the column vector $\underline{I}_g = 0$. With these values for \underline{V}_g and \underline{I}_g we arrive at a circuit representation which is precisely the one given in the equations 2.20 and 2.21 in [5] for a description in terms of a scattered voltage which corresponds precisely to what we are doing here.

VII. NUMERICAL EXAMPLE

Consider the coupled microstrip lines on a lossy substrate of Fig. 2. In [2] the transmission line parameters \underline{C} , \underline{L} , \underline{R} , and \underline{G} in (5) for this structure were calculated up to 100 GHz. In

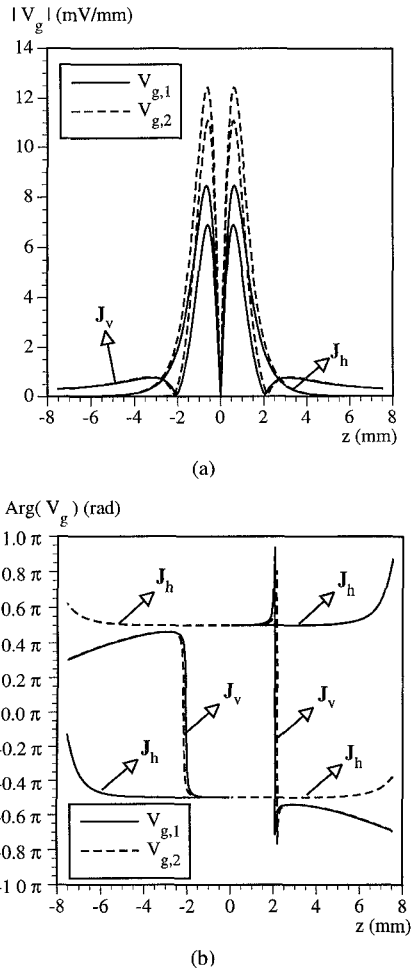


Fig. 3. (a) Amplitude and (b) phase of the distributed voltage sources $V_{g,1}$ and $V_{g,2}$ due to a vertical dipole $\mathbf{J}_v = J\mathbf{u}_y \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ and a horizontal dipole $\mathbf{J}_h = J\mathbf{u}_x \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ above the structure of Fig. 2.

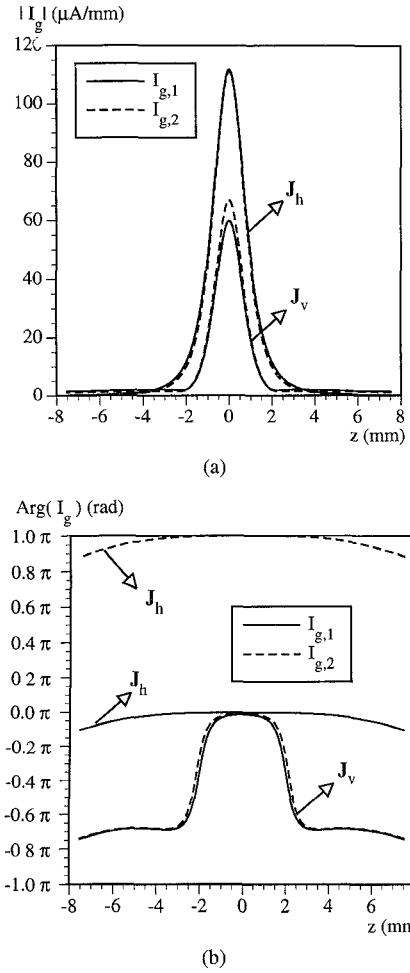


Fig. 4. (a) Amplitude and (b) phase of the distributed current sources $I_{g,1}$ and $I_{g,2}$ due to a vertical dipole $\mathbf{J}_v = J\mathbf{u}_y \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ and a horizontal dipole $\mathbf{J}_h = J\mathbf{u}_x \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ above the structure of Fig. 2.

this example we will work at a frequency of 10 GHz. The transmission line parameters at 10 GHz are given by

$$\begin{aligned} \underline{C} &= \begin{pmatrix} 242.7 & -9.978 \\ -9.978 & 167.8 \end{pmatrix} \text{ pF/m} \\ \underline{L} &= \begin{pmatrix} 302.4 & 68.82 \\ 68.82 & 386.6 \end{pmatrix} \text{ nH/m} \\ \underline{R} &= \begin{pmatrix} 4.45 & 2.95 \\ 2.95 & 4.56 \end{pmatrix} \Omega/\text{m} \\ \underline{G} &= \begin{pmatrix} 71.80 & -2.7 \\ -2.7 & 48.42 \end{pmatrix} \text{ mS/m.} \end{aligned} \quad (35)$$

We will excite the structure by a vertical electrical dipole $\mathbf{J}_v = J\mathbf{u}_y \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ (as shown on Fig. 2) and a horizontal electrical dipole $\mathbf{J}_h = J\mathbf{u}_x \delta(x) \delta(z) \delta(y - 2 \text{ mm})$ above the structure. The amplitude J of both dipoles is 1 mA/mm. In order to calculate the z -dependent, distributed sources $\underline{V}_g(z)$ and $\underline{I}_g(z)$ with (25) we first need the modal tangential magnetic field components at the surfaces of the strips or in other words we need the modal surface current distributions [cf. (16)] on the strips. These modal surface current densities were found with the technique described in [9]. In (25) on the other hand, we need the electric field \mathbf{e}^i generated by

the dipoles above the structure of Fig. 2 without the strips. This field is calculated easily with a classical spectral domain technique to incorporate the layered medium (see for example Chapter 6 in [4]). Figs. 3 and 4, respectively, show the complex voltage sources $V_{g,1}(z)$ (full lines), $V_{g,2}(z)$ (dashed lines) and complex current sources $I_{g,1}(z)$ (full lines), $I_{g,2}(z)$ (dashed lines) for the vertical dipole \mathbf{J}_v and the horizontal dipole \mathbf{J}_h .

The results for the current sources are the easiest ones to interpret. From Fig. 4(a) it is clear that $I_{g,1}(z)$ and $I_{g,2}(z)$ are symmetric but with an about two times larger amplitude for the horizontal dipole. Looking at Fig. 4(b) shows that for the horizontal dipole the sign of $I_{g,1}(z)$ and $I_{g,2}(z)$ is different as the respective phases are close to 0 and π . For the vertical dipole the phase of $I_{g,1}(z)$ and $I_{g,2}(z)$ almost coincides but shows a rapid variation between $z = -3 \text{ mm}$ and $z = 3 \text{ mm}$.

The voltage source results have an asymmetric nature with respect to the y -axis with again a larger amplitude for the horizontal dipole case but only by a factor of 1.1. Here again, the phase results for the vertical dipole vary rather quickly for $-3 \text{ mm} < z < 3 \text{ mm}$ while for the horizontal dipole there is again a difference of about π in the phase results for both

conductors. Remark that in all cases, due to the asymmetry, the phase suffers a jump discontinuity by an amount of π at $z = 0$ mm, i.e., at the place where the amplitude becomes zero.

VIII. CONCLUSION

We have constructed a rigorous high-frequency circuit model for the impinging of external electromagnetic waves on waveguide structures and in particular on multiconductor transmission lines. The model consists of distributed current and voltage sources which are expressed as line integrals around the conductors of the modal field distributions and the impinging fields. Finally, to illustrate our approach, a numerical circuit model was constructed for electric dipoles above two coupled thick lossy microstrip lines.

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